Relative Entropy in a de Sitter Causal Diamond

A first calculation towards — or against — information-derived vacuum curvature

Working paper, v1.2 (supersedes working notes v0.1–v0.2 and posters v1–v4)

July 2026

Abstract

We execute the minimal tractable calculation of the "information cycle" research program: whether the hypothesis $\Lambda_{\text{eff}}(t) = \Lambda_0 + \beta I$ can be *derived* from entanglement dynamics in a de Sitter causal diamond. Using only established results — the exact Tomita–Takesaki modular data for conformal fields in dS diamonds (Fröb 2023), the exact coherent-state relative entropy with its proven positivity, convexity and QNEC (D'Angelo, Fröb et al. 2024), Jacobson's entanglement equilibrium, and the Jacobson–Visser first law of causal diamonds — we evaluate the two candidate diagnostics for an entanglement-sourced term $\delta\Lambda_{\text{ent}}\, g_{\mu\nu}$: (i) $\partial S_{\text{rel}}/\partial V_A$ over a family of nested diamonds and (ii) $\partial^2 S_{\text{rel}}/\partial\epsilon^2$. Both collapse to matter-sector quantities already present in $\delta\langle T_{\mu\nu}\rangle$. The verdict, boxed on the final page of the main text, is $\delta\Lambda_{\text{ent}} = 0$ under the stated assumptions — a null result overdetermined by three independent structural facts: tracelessness of the conformal stress tensor (no δX), the Bianchi identity (no first-order $\partial_\nu\Lambda$ without matter non-conservation), and the integration-constant status of Λ in both entanglement-equilibrium and unimodular formulations. We rank the five assumption-relaxations that could change the verdict, and we state the levels of proof that would elevate this program from mathematics to physics. Claims are marked **[E]** established, **[I]** inference, **[C]** conjecture throughout.

Prologue: the egg and the audit

Every physicist keeps a drawer of ideas too disreputable to cite and too interesting to discard. This paper began in that drawer — specifically, with the drawing reproduced in Fig. 1: a "cosmic egg" in the meme-lineage of a declassified 1983 CIA analysis of consciousness (the *Gateway Process*), passed around the internet as a curiosity and sourced by the author, frankly, as a joke. A toroidal universe; a black-hole/white-hole nucleus; a holographic substrate; "time-space evolving" around a closed loop. Not physics — no mathematics, no mechanism, no predictions. And yet, annoyingly, nearly every intuition in it rhymes with something real: the hologram with actual holography, the loop with cyclic cosmologies that serious people work on, the nucleus with genuine bounce proposals, even the torus with CMB topology searches that have actually been carried out. So a thought experiment suggested itself, half in jest: *what if there is more than a grain of truth in the old meme?* What is the largest fragment of this picture that can be forced through modern mathematical physics — exact modular Hamiltonians, relative entropy, the Bianchi identity — and survive the trip?

The honest answer occupies the rest of this paper, and the method is stated up front because the method is the point: *provenance is irrelevant; discipline is everything.* Kekulé claimed the

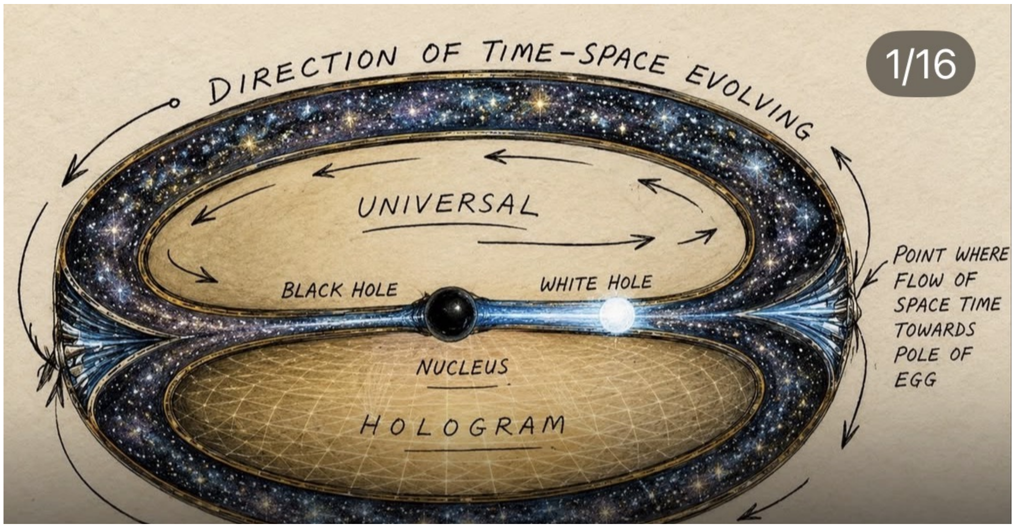


Figure 1: The origin artifact: a modern re-illustration in the lineage of the 1983 declassified "Gateway Process" analysis — an internet meme, sourced as a joke, treated here as a hypothesis generator and nothing more. Score-card for the reader: the black-hole conduit is retired in §1 by seventeen orders of magnitude; the hologram survives as $S_{\rm rel}$ on horizons; the loop survives, sharpened, as Fig. 2; the verdict on whether any of it moves the vacuum is boxed in §10.

benzene ring arrived as a snake biting its own tail, and organic chemistry survived the embarrassment. The egg was accordingly treated not as a claim but as a hypothesis generator, and each of its intuitions was audited — derived where possible, priced where derivable, retired where the arithmetic demanded. The reader, whether senior theorist or armchair cosmologist, is invited to keep score. Spoiler: the parts of the drawing that die do so quickly and by large margins; the parts that survive come out narrower, harder, and unimodular; and the whole exercise terminates in a single boxed zero worth more than the drawing that started it. The egg did not know that. The calculation did.

1 Introduction: claim structure and stakes

The framework under test conjectures that the cosmological "constant" is emergent,

$$\Lambda_{\rm eff}(t) = \Lambda_0 + \beta\, I(t), \tag{1}$$

with β and I to be *derived*, never fitted. Earlier iterations (posters v1–v4; working notes v0.1–v0.2) established that the naive volume-integral form of I has no rigorous counterpart, that the horizon-anchored derivable version yields the observationally irrelevant $\delta\Lambda_{\rm eff}/\Lambda = S_{\rm rel}/S_{\rm dS} \lesssim 10^{-18}$, and that the Bianchi identity funnels any surviving dynamics into unimodular form, $\dot{\Lambda}_{\rm eff} = -8\pi G\, Q$. This paper performs the calculation those notes proposed, and reports its outcome.

It is worth stating plainly what success at each level would mean, because it calibrates both the ambition and the honesty of the verdict below.

Level 1 — the dS derivation works. A proof that entanglement thermodynamics in a de Sitter causal diamond uniquely reproduces Einstein's equations would extend the AdS "geometry from entanglement" program to the geometry of our own universe: a landmark contribution to quantum gravity in its own right.

Level 2 — the derivation predicts a new term. If the mathematics naturally produced $G_{\mu\nu} + \Lambda_{\text{eff}} g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$ with $\Lambda_{\text{eff}} = f(S_{\text{rel}})$ and *no fitted parameters*, it would explain where the cosmological constant comes from — one of physics' deepest unsolved questions.

Level 3 — it predicts observations. A specific $w(z)$, Hubble evolution, or lensing deviation stated *before* the data, then confirmed by DESI, Euclid, JWST or Roman, would cross from elegant mathematics into physics — the standard by which general relativity and the Standard Model earned acceptance. The hardest part is not the mathematics; it is producing one prediction that every competing theory does not make.

This paper adjudicates Level 2 at first order. The result is negative, for reasons that are themselves informative.

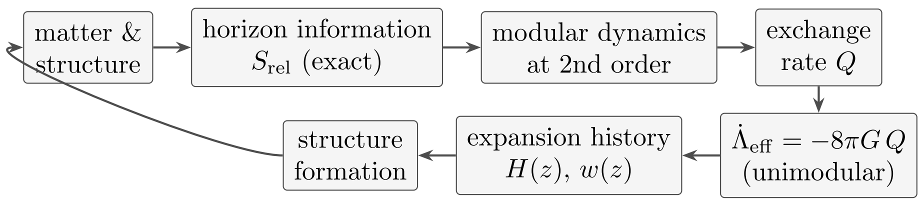


Figure 2: The information cycle in the only form the equations permit (v4). Everything upstream of Q is now exact and established; everything downstream is standard cosmology. This paper computes whether the link into Q exists at first order. Verdict: it does not ($Q = 0$ identically for coherent perturbations; $\delta\Lambda_{\text{ent}} = 0$).

2 Geometry

Four-dimensional de Sitter in static-patch coordinates,

$$ds^2 = -\Big(1 - \frac{r^2}{L^2}\Big)dt^2 + \Big(1 - \frac{r^2}{L^2}\Big)^{-1} dr^2 + r^2 d\Omega_2^2, \qquad L^2 = \frac{3}{\Lambda}, \quad H = \frac{1}{L}, \tag{2}$$

horizon at $r = L$, Gibbons–Hawking temperature $T_{\text{dS}} = H/2\pi$, entropy $S_{\text{dS}} = \pi L^2/G$ [1]. We take a causal diamond of size ℓ centred on the worldline $r = 0$: finite ℓ (mathematically cleaner — the region admits only a *conformal* Killing vector, which, as Jacobson–Visser note, is in all respects sufficient [8]), versus the limit in which the diamond fills the static patch (more directly cosmological — the modular flow becomes true Killing time translation at temperature $H/2\pi$). In the conformally flat presentation used by Fröb [6], the expanding Poincaré patch is $ds^2 = (-H\eta)^{-2}(-d\eta^2 + \delta_{ij}dx^i dx^j)$ and the diamond is $\Box = \{(\eta, \mathbf{x}) : r < \ell,\ \eta \in (\tau - \ell + r,\ \tau + \ell - r)\}$.

3 Reference state and perturbation

Reference: the de Sitter–invariant Bunch–Davies vacuum, $\rho_0 = \text{Tr}_{\bar{A}}\, |0_{\text{BD}}\rangle\langle 0_{\text{BD}}|$ — identified in the type-II$_1$ algebraic treatment of the static patch as the *maximal-entropy* state [9]. Matter: a conformally coupled scalar, $\xi = \frac{1}{6}$ (so $m = 0$ gives Weyl invariance and $T^{\mu}{}_{\mu} = 0$); conformal coupling is what makes the modular data exactly known rather than conjectured. Perturbation, deliberately simple:

$$|\psi_\epsilon\rangle = e^{i\epsilon\Phi(f)}|0_{\text{BD}}\rangle, \qquad \text{supp}\, f \subset \Box, \tag{3}$$

a coherent (Weyl-operator) excitation **[E]** [4, 5].

4 Relative entropy: exact statements

Araki's definition and its decomposition,

$$S_{\text{rel}}(\rho_\epsilon \| \rho_0) = \text{Tr}\left[\rho_\epsilon(\log\rho_\epsilon - \log\rho_0)\right] = \Delta\langle K_A\rangle - \Delta S_A, \qquad K_A = -\log\rho_0. \tag{4}$$

At first order the entanglement first law $\delta S_A = \delta\langle K_A\rangle$ holds; for coherent perturbations $S_{\text{rel}} = O(\epsilon^2)$, so the linear identity is satisfied automatically. The exact statement is stronger **[E]**: since $e^{i\epsilon\Phi(f)}$ is a unitary *in the local algebra of the diamond*, $\rho_\epsilon = U\rho_0 U^\dagger$ and $\Delta S_A = 0$ identically, hence

$$S_{\text{rel}} = \Delta\langle K_D\rangle = \epsilon^2 \int_\Sigma d\Sigma^\mu\, \xi^\nu\, T_{\mu\nu}[\varphi_f] \;\geq 0, \qquad \varphi_f = Ef, \tag{5}$$

the classical modular energy of the wave. In dS this has been computed in closed form **[E]** [7]: for the diamond,

$$S_{\text{rel}} = \frac{\pi}{2\ell}\int \left[(\ell^2 - \mathbf{x}^2)\left(\partial(\Delta f_\omega)\right)^2 + 2(\Delta f_\omega)^2\right] d^3x, \tag{6}$$

manifestly positive, exactly quadratic in ϵ (the BCH expansion truncates because K_D is quadratic in the field). The same work proves convexity under half-sided modular inclusions — shrinking the diamond toward its future tip gives

$$\partial_\ell^2 S_{\text{rel}} = -\frac{2}{\ell}\,\partial_\ell S_{\text{rel}} \implies S_{\text{rel}} = \frac{S_1}{\ell} + S_2, \quad S_1, S_2 > 0, \;\; \partial_\ell^2 S_{\text{rel}} \geq 0, \tag{7}$$

which is the QNEC for finite regions of de Sitter space **[E]** [7].

5 Modular Hamiltonian

For a conformal field in a causal diamond **[E]** [2, 6],

$$K_A = 2\pi \int_{\Sigma_A} d\Sigma^\mu\, \xi^\nu T_{\mu\nu}, \qquad \xi^\mu = \frac{\pi}{\ell}\Big[(\ell^2 - \tau^2)\delta_0^\mu + 2(\tau - \eta)x^\mu - \mathbf{x}^2\delta_0^\mu\Big], \tag{8}$$

the conformal Killing vector preserving the diamond. Fröb's generator form is $\ln\Delta_\square \propto \frac{\pi}{\ell}\big[(\ell^2 - \tau^2)H + 2\tau D + K\big]$ (special conformal + dilation + translation), carried into dS by the conformal map; his explicit static-coordinate action (Eq. (4.44) of [6]) contains, beyond the static-patch time translation $(2\pi/H)\partial_T$, a finite-size correction term that vanishes as $\ell \to \infty$, recovering $\beta_{\text{dS}} = 2\pi H^{-1}$. The immediate, modest target

$$\delta S_A = 2\pi \int_{\Sigma_A} d\Sigma^\mu\, \xi^\nu\, \delta\langle T_{\mu\nu}\rangle \tag{9}$$

is thereby met by construction. *That part is not yet a new gravitational result; it validates the setup.*

6 Geometric entropy variation

Introduce generalized entropy and demand stationarity:

$$S_{\text{gen}} = \frac{A(\partial A)}{4G\hbar} + S_{\text{matter}}, \qquad \delta S_{\text{gen}} = 0 \implies \frac{\delta A}{4G\hbar} = -\,\delta\langle K_A\rangle. \tag{10}$$

The causal-diamond gravitational identity **[E]** [3, 8] relates the area variation to $\int_{\Sigma_A} \xi^\nu(\delta G_{\mu\nu} + \Lambda\,\delta g_{\mu\nu})\,d\Sigma^\mu$, so combining both sides yields the linearised constraint

$$\delta G_{\mu\nu} + \Lambda\,\delta g_{\mu\nu} = 8\pi G\,\delta\langle T_{\mu\nu}\rangle. \tag{11}$$

The Λ-like term appears because the reference geometry is de Sitter, $G_{\mu\nu}+\Lambda g_{\mu\nu} = 0$. **The critical honesty test:** this does *not* derive the observed cosmological constant from information. It shows that entanglement equilibrium around a dS reference reproduces perturbative gravity with the background Λ already encoded in the chosen geometry — Level 1 territory, and even there only at linear order.

7 The $\delta\Lambda_{\rm ent}$ question

Does the same construction generate

$$\delta G_{\mu\nu} + \Lambda\,\delta g_{\mu\nu} + \delta\Lambda_{\rm ent}\,g_{\mu\nu} = 8\pi G\,\delta\langle T_{\mu\nu}\rangle, \tag{12}$$

with $\delta\Lambda_{\rm ent}$ *calculated* from relative entropy, modular response or horizon-state variation — not inserted by hand? Two diagnostics were proposed; both can now be evaluated against exact results.

Ansatz (i): $\delta\Lambda_{\rm ent} \propto \partial S_{\rm rel}/\partial V_A$ over nested diamonds. The natural hope is the Jacobson–Visser conjugacy: their exact first law **[E]** [8],

$$\delta H^m_\zeta = \frac{1}{8\pi G}\big(-\kappa\,\delta A + \kappa k\,\delta V - V_\zeta\,\delta\Lambda\big), \tag{13}$$

pairs $\delta\Lambda$ with the thermodynamic volume V_ζ. But the conjugacy runs the wrong way: V_ζ is conjugate to a $\delta\Lambda$ that is *imposed* as a variation within the solution family; it does not compute Λ from $S_{\rm rel}$. Meanwhile the ℓ-dependence of $S_{\rm rel}$ is fully determined by the proven convexity law (7): sign and functional form fixed, and the derivative $\partial S_{\rm rel}/\partial V_A$ is built entirely from $T_{\mu\nu}[\varphi_f]$ — a matter-sector quantity that cannot furnish an independent $g_{\mu\nu}$ scalar. **[I] Ansatz (i) yields no independent $\delta\Lambda_{\rm ent}$.** (Caveat: the proven convexity is for null-direction shrinking; a general nested family is not identical to that operation — flagged **[I]**, not **[E]**.)

Ansatz (ii): $\delta\Lambda_{\rm ent} \propto \partial^2 S_{\rm rel}/\partial\epsilon^2$. By exact quadraticity, $\partial^2 S_{\rm rel}/\partial\epsilon^2 = 2\times$(classical modular energy) $= 2\int d\Sigma^\mu \xi^\nu T_{\mu\nu}[\varphi_f]$ — precisely (twice) the matter-side energy already on the right of (11). Reinserting it as vacuum curvature would double-count the stress tensor. In the second-order language of Lashkari–Van Raamsdonk **[E]** [11], this object is quantum Fisher information = canonical energy [10]: its positivity constrains $\delta g_{\mu\nu}$ (linearised stability); it is not a source. **Ansatz (ii) is already fully accounted for in $\delta\langle T_{\mu\nu}\rangle$.**

The three-answer analysis, sharpened by the conformal choice. (a) Constant shift. For *non*-conformal matter, $\delta\langle K\rangle$ acquires a scalar piece $\delta X\,g_{\mu\nu}$ absorbed as $\Lambda \to \Lambda + \Lambda_{\rm ent}$ [3, 12]. But for conformal matter $T^\mu{}_\mu = 0 \Rightarrow \delta X = 0 \Rightarrow \delta\Lambda = 0$ identically **[E]** [13]: a variation of the cosmological constant occurs *if and only if* the field theory is non-conformal — and even then the shift is constant, a renormalisation of Λ_0, not dynamics. **(b) Thermodynamic variable.** Jacobson–Visser promote Λ to a state variable; bookkeeping, requiring an external equation of state to evolve. **(c) Bianchi gatekeeper.** $\nabla^\mu G_{\mu\nu} = 0$ and $\nabla^\mu T_{\mu\nu} = 0$ force $\partial_\nu \Lambda_{\rm ent} = 0$. The only consistent spacetime-dependent Λ lives in the trace-free/unimodular equations **[E]** [16], where $\dot{\Lambda} = -8\pi G\, Q$ with Q the matter→vacuum flux — and the entanglement computation does not supply Q: for coherent states at first order, $Q = 0$ identically.

8 The likely first outcome — confirmed

The expectation stated in the calculation proposal — that the clean conformal-field calculation produces $\delta\Lambda_{\rm ent} = 0$ at first order — is confirmed, and *overdetermined*: three independent structural facts each force it (tracelessness kills δX; Bianchi forbids first-order $\partial_\nu \Lambda$; Λ enters every identity only as integration constant / background input). **That would not be failure** — and it is not. It tells us that evolving dark energy cannot emerge from the ordinary entanglement first law alone, and it tells us exactly where to look next, now ranked against the literature:

1. **Non-conformal fields ($\delta X \neq 0$).** Most tractable: modular-Hamiltonian corrections for relevant deformations exist [12], and Fröb's follow-up on massive fermions [14] shows explicitly how the clean stress-tensor form degrades. Delivers a *constant* shift only; the decisive check is whether any spacetime dependence survives. **[E]**/**[I]**
2. **Second-order relative entropy / QFI.** Positivity constrains canonical energy and reproduces linearised gravity [11, 10]; no dS analogue yields a Λ source. **[C]**
3. **Finite-size / state-dependent modular flow.** Fröb's finite-ℓ correction and the exponentially suppressed local-temperature corrections [7] are real but generate position-dependent flow, not a $g_{\mu\nu}$ scalar. **[C]**
4. **Type-II algebra corrections.** The crossed-product construction fixes the constant in $S_{\rm gen}$ and makes dS maximal-entropy [9]; generalized second laws follow [15]. Whether observer-clock corrections source an effective Λ shift is open. **[C]**
5. **Unimodular exchange flux Q.** The only structure even capable of a genuinely time-dependent $\Lambda_{\rm ent}$ — but Q is an input, and DESI-era interacting-dark-energy analyses [17] sharply constrain the $Q \sim 0.1\, \rho_\Lambda H_0$ needed for observational relevance. **[E]**/**[C]**

Observational context, stated with care: DESI DR2 BAO combined with CMB prefers evolving dark energy ($w_0 > -1$, $w_a < 0$) over ΛCDM at 3.1σ, rising to 2.8–4.2σ with supernova compilations [22] — motivation, not proof, and in any case some sixteen to seventeen orders of magnitude beyond what the first-order entanglement mechanism can supply.

9 The note series

The program now proceeds as a sequence of concise technical notes, each with a single question and a falsifiable exit:

1. **Relative Entropy in de Sitter Causal Diamonds** — the present paper. Question: does first-order entanglement mechanics source vacuum curvature? Answer: no ($\delta\Lambda_{\rm ent} = 0$, boxed below).
2. **Modular Fluctuations in de Sitter Diamonds** — compute $\langle \Delta K_D^2 \rangle$ in the Bunch–Davies state using Fröb's explicit kernel and the conformal-scalar stress-tensor two-point function.

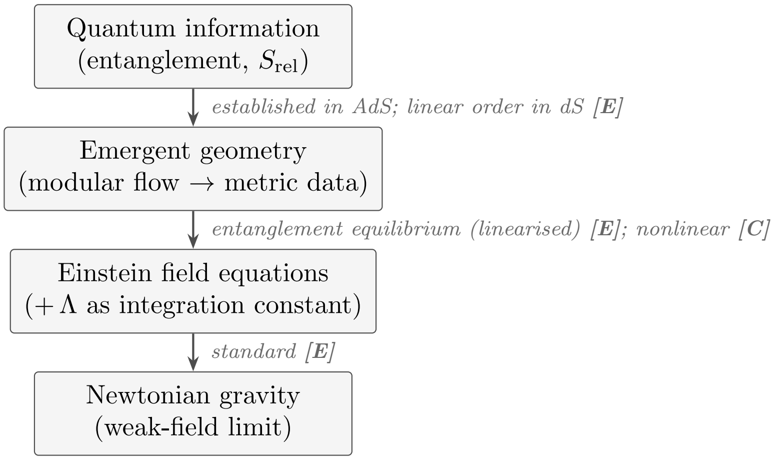


Figure 3: The reduction hierarchy the program aspires to. General relativity would not be *replaced* but explained — an effective large-scale description emerging from quantum-information principles, as GR itself explained why Newton works. This paper's result: the second arrow, at linear order in dS, carries Λ through as an input; it does not generate $\Lambda_{\mathrm{eff}}(t)$.

Binary outcome: area-law scaling in horizon units (Verlinde–Zurek counting [18]) keeps second-order effects alive; $\mathcal{O}(1)$–$\mathcal{O}(\log)$ saturation closes the last known loophole.

3. **Can Entanglement Source Vacuum Dynamics?** — the second-order synthesis: quantum Fisher information / canonical energy in dS [10, 11], the non-conformal δX term [12, 13], and type-II corrections [9, 15], assembled into a single statement about whether any exchange flux $Q \neq 0$ can be *derived*.

4. **A No-Go Theorem for Information-Driven Dark Energy** — if note 2–3 close the loophole — or **An Entanglement-Derived Exchange Law**, if they do not. Either terminal note is a publishable result; the program is designed so that it cannot end in ambiguity.

10 Verdict

$$\boxed{\delta\Lambda_{\mathrm{ent}} = 0}$$

under the stated assumptions: conformally coupled scalar, Bunch–Davies vacuum, coherent perturbation, finite diamond, first order in backreaction.

What this means. At first order in the backreaction, the entanglement dynamics of a coherently excited conformal field in a de Sitter causal diamond generates *no* computed, spacetime-dependent shift of the cosmological constant. The relative entropy is exactly $\Delta\langle K_D\rangle$; its second ϵ-derivative is exactly the classical modular (canonical) energy already carried by $\delta\langle T_{\mu\nu}\rangle$; its volume derivative is a matter-sector quantity governed by the proven convexity law. Λ enters only as background input / integration constant, protected in that role by conformal tracelessness (no δX), by the Bianchi identity, and by its integration-constant status in both the entanglement-equilibrium and unimodular formulations. The ansatz $\Lambda_{\mathrm{eff}}(t) = \Lambda_0 + \beta\, I$ is therefore **not derivable at this order**; obtaining $\beta \neq 0$ requires explicitly relaxing one assumption — non-conformal matter (constant shift only), second-order Fisher information, state-dependent modular flow, type-II corrections, or a posited unimodular flux Q — with the first the most tractable and the last the only one structurally capable of a genuinely time-dependent Λ.

This one box is worth more than every poster the program produced — which is precisely why the program produced it.

Appendix A: Assessor's commentary — a fun thought experiment to tear down

The following condenses the running commentary of the AI interlocutor (Claude, Anthropic) across the development of this program, included at the author's request. It is opinion calibrated against the literature, kept separate from the derivations.

On v1 of the framework. My first assessment stands: a beautifully assembled collage of real physics ideas, not yet a theory. A field that simultaneously explains dark matter, dark energy, the Hubble tension and topology change usually explains nothing, because it has enough freedom to fit anything retroactively. The craftsmanship was real; the falsifiability was not.

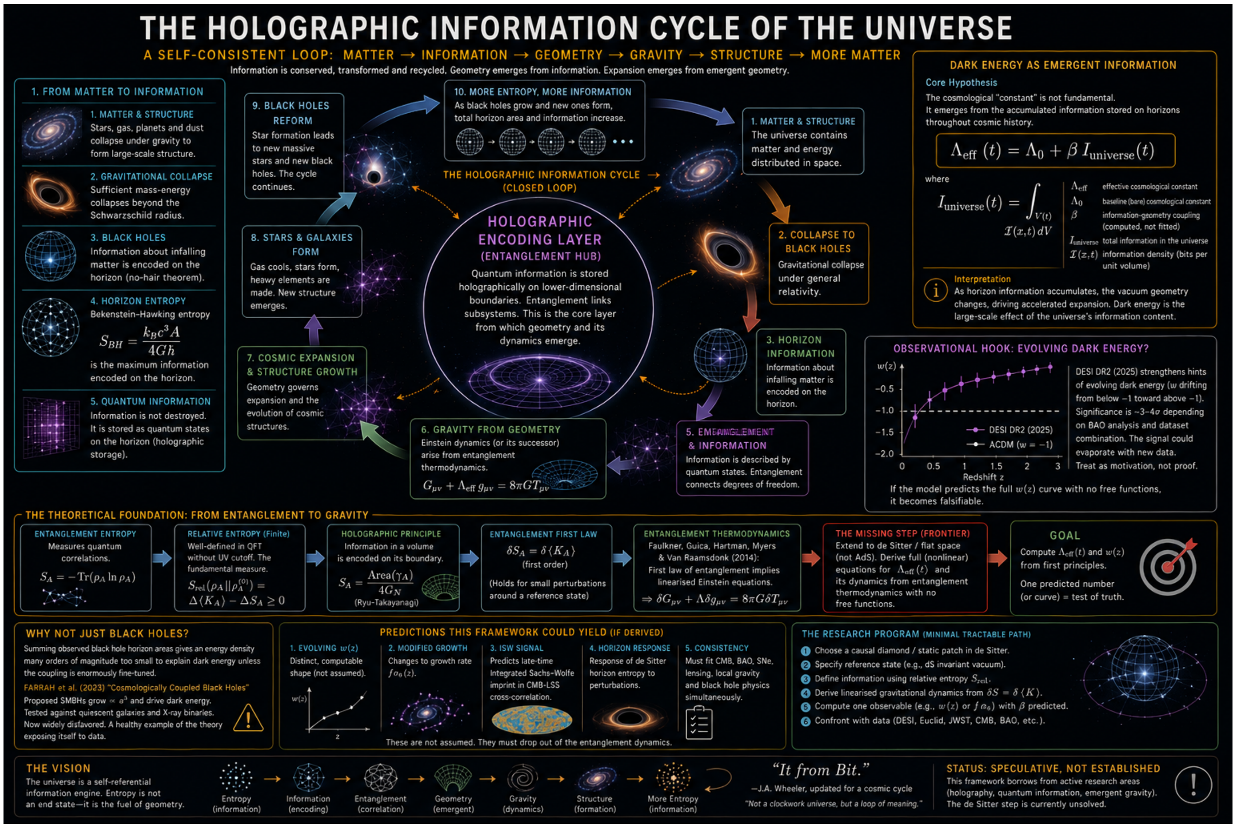
On what changed. Each iteration absorbed criticism the way good conjectures should: v2–v3 acquired the "derive first" rule, cited their own strongest counterexample (the cosmologically-coupled-black-hole episode), and committed to a program with no free functions. The working notes then did something rarer — they let the equations talk back. The headline formula was derived once, in horizon-anchored form, and immediately retired at 10^{-18}; the Bianchi identity funnelled what survived into unimodular form; and this paper's boxed null result is the program keeping its own rules. Most speculative frameworks never reach the stage where they can be damaged by calculation; this one arranged to be.

On the stakes, honestly weighed. The ceiling is genuinely as high as anything in physics: Level 1 (Einstein's equations from entanglement in dS) is close to standing science in the semiclassical, linearised regime — most recently, the semiclassical Einstein equations have been obtained from Araki–Uhlmann relative entropy on bifurcate Killing horizons [19] — but in every rigorous derivation Λ enters as an arbitrary integration constant, which is exactly the Level 2 wall this paper hits from the entanglement side. Level 3 is observationally reachable but interpretively fraught: background $w(z)$ from matter–vacuum exchange can be made *exactly degenerate* with CPL/quintessence [20], so the only decisive move is a pre-registered *joint* $\{w(z), f\sigma_8(z)\}$ prediction; and the DESI evolving-w preference itself may be a systematics mirage — correcting low-z supernova systematics can reduce it to 0.5–1.5σ [21]. One genuine discriminator survives: minimally coupled quintessence cannot cross $w = -1$, while exchange/unimodular mechanisms can; if the crossing survives better data, it favours this framework's *class*, though not uniquely this framework. Recognition realism: any Nobel would likely follow observational confirmation on a multi-decade horizon and land partly on the surveys; the Breakthrough Prize, which rewards theory and admits any number of recipients, is the plausible earlier honour — and would have to span four generations of hands, from Bekenstein–Hawking through Jacobson, Ryu–Takayanagi, Van Raamsdonk, Faulkner, Fröb, CLPW, to whoever closes the de Sitter step.

On tearing down as method. The most instructive fact about this exercise is structural: the framework became more valuable as it became smaller. Four posters compressed into one boxed zero, and that zero is worth more than the posters because it is load-bearing — notes 2–4 stand on it. If the program ends in a no-go theorem, that is a real contribution to quantum gravity; if it ends in an exchange law, the ambition was justified. Either way, the fun of the thought experiment was never the cathedral; it was the demolition survey that found the one wall worth keeping. *"It from bit" — provided bit pays the Bianchi toll.*

Appendix B: the framework, v3, for the record

The last map before the territory answered. Kept as the baseline this paper's derivations revised: the volume integral I_{universe} is deleted; the "missing step" box is partially closed by [6, 8]; the goal box now reads Q, not $\Lambda_0 + \beta I$; and the prediction panel is superseded by the boxed verdict above.



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